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SUBSTANTIATION OF THE METHOD OF INTEGRATED GROUP UNIFICATION OF MACHINE AND APPLIANCE DESIGNS

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Об'єктом дослідження є групова уніфікація конструкцій технологічних машин та приладів. Уніфікація є одним з важливих важелів підвищення ефективності виробництва і експлуатації вузлів (деталей), що знижує собівартість їх виготовлення і ремонту. Також уніфікація є підсистемою стандартизації, що істотно підсилює інтерес до її дослідження та впровадження.

Однією з проблем розвитку групової уніфікації конструкцій є відсутність достатньої теоретичної бази, а роботи в напрямку уніфікації нерідко зводяться до симпліфікації. Зазначене є причиною зниження ефективності виробництва через сповільнення процесу створення та впровадження уніфікованих конструкції при сталих темпах росту номенклатури вузлів (деталей), оснащення та інструменту.

Запропоновано підхід, в основу якого покладена гіпотеза про можливість знаходження критеріїв (формул), які дозволять апріорно оцінити відповідність структури конструкцій встановленим рівням уніфікації. А також виявити закономірності та зазначити методи оптимізації структури конструкцій шляхом адаптації до технологічного оснащення. Реалізація такого підходу здійснювалася шляхом використання аксіоматичної теорії, законів композиції, теорії груп та символічної логіки.

В результаті дослідження дано визначення первинного елементу та представлено методику його побудови, виведено формули уніфікованих деталей та сформульовано теорему уніфікації структури конструкцій вузлів (деталей). Розглянуто особливості комплексної уніфікації груп деталей та оснащення для їх виготовлення.

Отримані результати досліджень дозволять удосконалити інтелектуальний конструкторський процес і сприятимуть широкому використанню систем автоматичного проектування технологічних конструкцій. Результати дослідження є цікавими:

– для конструкторів підприємств при створенні закритих баз даних уніфікованих деталей (вузлів), що значно скоротить терміни розробки та впровадження нових виробів, підвищить їх ефективність;

– для користувачів програмного забезпечення при створенні доступних відкритих баз даних уніфікованих деталей(вузлів), що мають за мету приховану рекламу та стимулювання продажів уніфікованих виробів

Ключові слова: первинний елемент, теорія груп, теорема уніфікації, комплексна групова уніфікація

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1. Introduction

Unification of machine and appliance designs is one of important means for improving efficiency of their production and operation. Unification has a significant impact on reducing the cost of production and repair and is closely related to manufacturability of products. Besides, unification is a standardization subsystem greatly attracting attention to its study and implementation.

Eventually, creation of computer aided design (CAD) systems did not contribute to a departure from static forms. The process of creating unified products has no enough scientific principles and the unification efforts are often reduced to a mere simplification. This is the cause of slow creation and implementation of unified designs while the rate of growth of nomenclature of parts and assemblies, rigging and tools does not decrease. This, in turn, has a negative impact on production efficiency.

Presence or occurrence of excess considered until recently as a necessary or acceptable part of unification and had its impact as well. For example, any appliance set was placed in one housing whose dimensions were taken proceeding from the highest internal saturation with assemblies and parts. Therefore, housing of one type of the appliance set was filled to the maximum and the other was half-filled, that is, its size and, consequently, cost of materials were excessive. This approach was justified by reduction in manufacturing costs due to the use of relatively simple high-performance process equipment, smaller variety and number of equipment units. At some stage of technology development, this trend was acceptable but the excessive consumption of materials should be excluded in today's production.

Therefore, study in the direction of development of theoretical base for unification of machine and appliance designs is relevant. It is necessary to solve the problems of obtaining quantitative criteria for a priori assessment of correspondence of machine and appliance designs to established levels of unification, derivation of designing rules, formalization of design unification and its combining with unification of process equipment.

2. Literature review and problem statement

In the case of group unification, construction of the primary element is of priority. Further derivation of formulas of unified parts and formulation of theorems of unification of structure of part (assembly) designs is based on correctness of construction of the primary element and establishment of regularities in the structures.

It is proposed in [1] to identify regularities in design structures by calculating predicates of the first order using mainly the rule of substitution and the modus ponens. According to [2], the obtained regularities should be taken into account when creating subsets by the type of subgroups selected from the set of parts belonging to a certain group. It is advisable to use the theory of sets, the theory of similarity [4] and the theory of groups [5] to realize the above. Application of these theories will significantly improve accuracy of the obtained results and is necessary from the standpoint of adhering to internal logic when constructing subsets. However, in addition to ensuring correctness of presentation, strict adherence to these theories is characterized by inertia of thinking.

The theorem of existence of designs facilitates departure from this inertia [6]. There are some reasons to use the theorem of existence of designs in designing. First, it may, contrary to the inertia of thinking, eliminate discussions (and thus save time) among designers on the possibility or impossibility of any particular designs since the fact of existence of all designs with real parameters is proved by the same theorem. Also, the theorem of existence somewhat changes the approach to design in the sense of its abstract representation. A concept of «design space» was introduced. It is an ideal space in which any known and unknown (though not removed from the design space given the abovementioned idealization) products whose existence is assumed in the mentioned space are located. As a result of this step, the conventional notion of design is replaced by an abstraction of «extraction of technical objects from the design space» which is convenient when formalizing the statement.

According to [7], the theory consists in terms and the concept is considered as an abstraction, that is, an imaginary reflection of essential features of the object. Advantage of this approach consists in that it takes into account original concepts and principles that express fundamental connections and relationships of the area under study which determine all other phenomena. However, excess of initial references is the major drawback that complicates the theory. According to [8], when building a rigorous theory, all initial references should be minimized and only substantiated terms should be used.

According to [9], formalization is a theoretical embodiment of logical completion of the theory. It connects structural elements of the theory starting with judgments, principles, concepts and ending with theorems, axioms, laws, and more. Advantages of using the formalization include void of polysemantic terms. There is no ambiguity of rules for constructing expressions for designs. Its drawback consists in that over-formalization weakens the theory.

The authors of [10] propose to apply formalization to designs in order to accelerate and improve quality of the designing process, making corresponding design documents, etc. They claim that one of the formalization methods consists in solving a wide range of design problems by switching them from the plane of «intelligent» design to the CAD. Implementation of «intelligent» design is of narrow-oriented nature and development of CAD for a wide range of design problems is associated with significant consumption of computer hardware resources and the cost of creating such a product.

However, the existing design assessment methods do not provide positive results when used at the development stage. For example, [10] refers to 13 major and additional indicators of processability. Besides, all of them must be determined in relation to the base product or base indicators. This approach reduces reliability of assessment. The base product is selected according to the achieved level and therefore the design cannot be objectively evaluated at the stage of development because of the use of data on labor content and cost price, i. e. a posteriori values in the used indicators. These values can only be obtained on completion of the design process, introduction in manufacture, determination of batch volumes and development of a technology. In order to improve the design quality and reduce time of creation of new designs, objective quantitative criteria of product evaluation (manufacturability, unification, maintainability) are needed as early as at the stage of their development.

According to study [11], laws, concepts, etc. are developed from the general principle underlying any theory. Principles are developed and refined only if the theory is formulated.

The authors of [12] recommend to build a theory based on the axiomatic method. Advantage of this method consists in ability to establish trueness of scientific claims, ensure rigor of construction by limiting redundancy. This method makes it possible to find such a system of axioms in which one can elaborate statements significant in the theory by means of logical constructions from axioms. The authors of [13] used axiomatic method to substantiate quantitative criteria for evaluating manufacturability of parts and assemblies. The results obtained from formalization of manufacturability criteria have made it possible to reduce the process of optimization of the design structure to a single algorithm with a high degree of automation. However, no studies have been conducted yet to unify groups of parts and technological equipment for their manufacture.

Logical and mathematical study of parts and the equipment and the process for their manufacture, that is, obtaining deduced knowledge of designing can serve as an effective way of finding rules of integrated unification of parts (assemblies) and the process equipment for their manufacture. However, numerous requirements to designing are contradictory, so it is advisable to arrange requirements using algebraic systems.

Features of constructing algebraic systems are disclosed in [14, 15]. Taking these features into account when formulating the theorem of unification of the design structures will make it possible to obtain criteria for a priori evaluation of parts (assemblies) and formalize unification of groups of part and equipment for their manufacture.

In recent years, considerable attention has been paid to the methods of modeling and optimization aimed at representation and improvement of individual event systems [16]. These methods can substantiate decision making by facilitating determination of the best combination in a combinatorial search space with stochastic variables.

One of the methods for rational organization of production is the group method based on defining and use of technological similarity of parts and assemblies. It is expedient to further develop it on the basis of the mathematical theory of calculations [17]. Several descriptive formalisms are presented in [17] with examples of their use and a theory that makes it possible to prove equivalence of calculations expressed in these formalisms. A number of formalisms for expressing computational functions and related objects are described. Advantages of using formalisms are pointed out in [6] where the theorem of existence of designs and its corollaries are presented with the help of the mathematical theory of calculations.

It is stated in [18] that the theory of groups is the most suitable mathematical apparatus for a formalized description of any technological system. By this method, the group can be represented as a set specified by generative elements and mappings which are closed relative to the associative operation in the presence of opposite elements and zero. However, this method has not been widely used in engineering. According to the authors of [19], use of the theory of groups is necessary at the current level of design development for integrated unification of the «product-technology» systems.

Analysis of the published data shows that, in theory, further development of unification requires deep formalization. Formalization can serve as a basis for widespread use of flexible process equipment and unified technological processes, reducing the nomenclature of designs, equipment and tools. This process will shorten time spent in development and introduction of new products and improve their efficiency.

3. The aim and objectives of the study

The study objective: development of a mathematical tool suitable for realization of comprehensive group unification of designs of machines and appliances and process equipment for their production.

To achieve this objective, the following tasks must be accomplished:

– justify construction of the primary element of group unification of assemblies (parts);

– derive formulas of unified parts using the theory of groups;

– formulate a theorem of unification of design structures using axioms and the theorem of existence of designs;

– demonstrate features of integrated unification of groups of parts and the equipment for their manufacture.

4. The methods used in the study of integrated unification of groups of parts and the equipment for their manufacture

During the scientific study,

– first-order predicates were determined by the theory of groups [5];

– the tasks of construction of the primary element as the basis of a part or a group of parts, creation of subsets by the type of subgroups chosen from the set of the parts belonging to the group were performed using the theory of sets [3], the theory of groups [5] and the theorem of existence of designs [6];

– the system-structural method [1] was used for analysis of parts (assemblies) of designs for the purpose of drawing up unification formulas;

– the law of composition was used for formalization and ordering of the unification process, algebraic operations were introduced, the notion of isomorphism and additive composition were used [20];

– the inductive method was used to prove the theorem of unification of the design structures which makes it possible to generalize the study results by a movement of thought from single to general (the logical method of study) [21];

– in order to ensure rigor of the theorem of unification of the design structures, an axiomatic method was used which is one of the ways of deductive construction of scientific theories [15];

– the system-structural method and the method of generalization and optimization of results were used to study comprehensive unification of groups of parts and equipment for their manufacture [22].

5. The results obtained in development of a mathematical tool for realization of integrated group unification of designs

5. 1. Substantiation of construction of the primary element in integrated group unification of designs

In group unification, the primary importance must be given to a correct construction of the primary element. Let us consider primary elements of the parts. To this end, represent structure of any part of the group. It is possible to specify its zones that have functional value φ (including an ergonomic and aesthetic value) as well as technological value t. The zones can be divided into the part elements and combined with each other. When constructing a part from a single element by repeating it, all kinds of zones are represented in that primary element. Auxiliary and transition zones are distinguished. What concerns technological zones, they can be technological bases, holes for orientation during processing, etc. Therefore, the primary element should be of such a size that it was possible to use it functionally as a part, and at the same time, it should be technological. Making the primary element equal to the parts solves the issue of its dimensions since all requirements relating to the part begin to apply to the primary element. Thus, a barrier for further practical sectioning of the primary element is set. However, this does not exclude the possibility of considering sections of the primary element and their changes pertaining compliance with requirements of unification (for example, introduction of restrictions on diameters of holes), manufacturability, etc.

If we represent the primary element α_0 and the parts designed from it such that α_0 is a reduction of the whole group to the primary element, then:

$$((f\colon \varphi\to\alpha_0)\wedge(f\colon t\to\alpha_0))=\alpha_0. \quad (1)$$

Expression (1) is the simplest formal entry of a primary element as a basis of a part or a group of parts. In the future, it is natural to switch to the part formula. A simple part is described by a simple formula. Complexity of the part can be judged from the formula and when optimizing the formula, the part is optimized.

If it is impossible to obtain one primary element for the whole group because of complexity of the parts entering the group, then it is advisable to form subsets in it by the type of subgroups. Such subsets are formed by separating from the set of parts belonging to the group sequences that will reduce to the element of each subgroup. Requirements to designing the subgroup element that provides the above reduction must be the same as to the primary element of the group.

Let us consider in more detail application of the theory of groups to unification of designs. The use of the theory of groups makes it possible to characterize unification in precise terms with respect to the structure of parts (assemblies). Application of the law of composition to the sets of parts (assemblies) and their elements, introduction of operations similar to algebraic ones, use of the concept of isomorphism, etc. make it possible to formalize and order the process of unification and improve its efficiency. Mathematically, a group is a set with one double (binary) operation or the law of composition. In this case, the law is associative, that is, it assigns a certain element of the same set to each pair of elements. Therefore, a group has neutral and opposite elements. Subgroup is a subset of the group and must meet requirements to maintaining of operations, availability of neutral and opposite elements. Taking the process of studying a set of design elements as an object, a structure is obtained in which use of formalized operations leads to essential applied results.

Formally, the group properties of designs are as follows. Let us consider the set K of any designs. The set is chosen in such a way that the law of composition is defined for it and it is associative whereby binary designs ensure commutativity of the law. As indicated above, there is an opposite element k for each element, that is, for each particular design $k \in K$. By analogy with an empty set, an «empty design» can be considered as a neutral element. In addition, all new designs resulting from the law of composition must belong to the group from which they were created as a result of an algebraic operation, that is:

$$\forall k \left(\begin{pmatrix} (k' \in G) \wedge \\ \wedge (k'' \in G) \wedge \\ \wedge (k' + k'' = k''') \end{pmatrix} \Rightarrow (k'' \in G) \right), \quad (2)$$

where k is the general designation of designs; k', k'' are any known designs; k''' is the new design; G is a group $k'' \in G$.

If these conditions are fulfilled, then the set of designs will be a group, commutative at that. Commutativity is defined as equality of a binary composition of conjugated designs k' and k'': $k' + k'' = k'' + k'$. In graphic images, opposing elements may exhibit ability to symmetry of designs which is an important factor in their optimization.

Let us consider an example of a primary element of any group of relatively simple bracket-type parts made of sheet material by punching (Fig. 1). The above part was chosen not only because of its design simplicity but also to describe as many as possible really existing parts of which punched ones make up more than a half. Take longitudinal axis of symmetry and linear dimensions, i. e. width l, two coordinates of the hole center $l/2$ as constants (non-changed parameters) in the primary element. The rest of dimensions (sheet thickness S, radius of rounding, hole diameter) can vary within the limits determined by equipment, dimensional dependences in the primary element, features of assembling in a part, etc. In particular, length of the section A (its width coincides with dimension l) can be set within $l/m...ml$ (m=1...n, that is, it is chosen from a natural series of numbers). The hole diameter may vary from zero (for example, punch removed) to a maximum value limited by a minimum bridge with width equal to the sheet thickness S. The punch can be calculated for punching holes appropriate for inner thread diameter which is most often performed for this type of parts. If it is necessary to increase diameter of the threaded hole, then drilling operation is performed. In this case, drilling radius varies from zero to a maximum of $l/2$. Instead of rounding, chamfer can be made (Fig. 1, *a*) which is technologically simpler given the dimension variation. In the absence of section A, the primary element is centrally symmetrical relative to the hole center.

Obviously, structural level of the primary element is the lowest and the primary element can independently perform function of the part or be a component of the part while having a repetition in a nonchanged form or vary within the above limits. Examples of combining parts from primary elements are shown in Fig. 2.

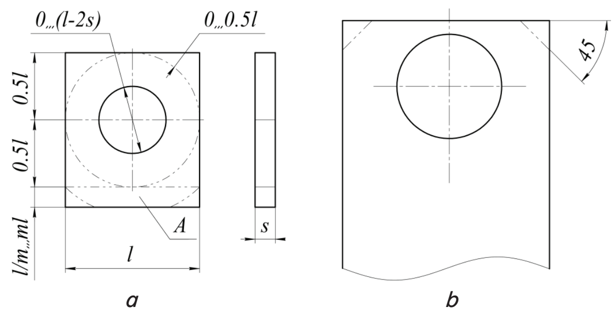

Fig. 1. Structural design of the primary element: *a* – with a radius of rounding; *b* – with a chamfer

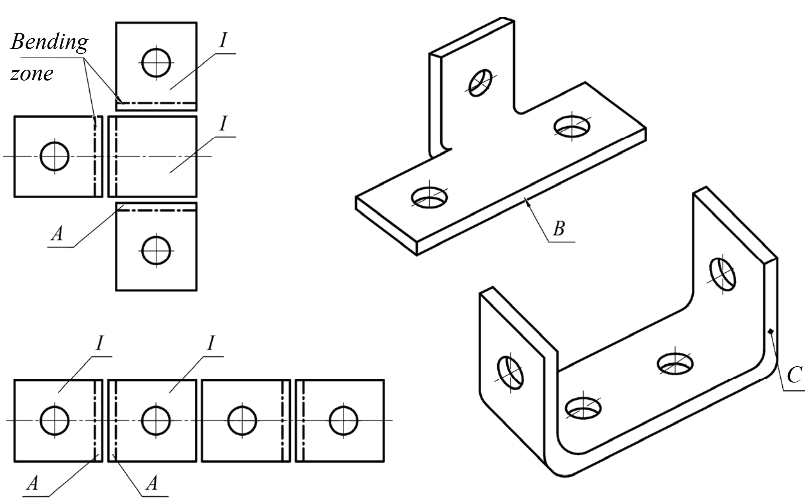

Fig. 2. Axonometric views of unified parts with sweeps in which primary elements are highlighted: *I* is the primary element; *A* is the zone of variable length to be bent; *B* is the bracket with one bent out shelf (corner); *C* is the bracket with two shelves

Using terminology of the theory of groups for this example, it should be noted that the primary element with all its dimensions fixed is the basic element of the part group and this group is cyclic, periodic and commutative with the centrally symmetrical primary element. This approach opens up the possibility of using apparatus of the theory of groups to formalize generation of unified parts and design them using the CAD system.

5. 2. Deriving formulas of unified parts (assemblies)

To proceed to the issue of deriving formulas of unified parts, let us make some preliminary observations. Abstract joints of the primary elements and the unified parts correspond in practice to obtaining of a complete part from a given material, such as by punching or turning or using, where it is necessary, joining operations such as welding or soldering. When designing unified parts, requirements of the theory of groups are taken in account. In particular, one must adhere to the rule saying that a part is composed of a pair of the group elements and each connected pair of elements generate an element that also belongs to that group. The formula describing a class of relatively simple parts having a longitudinal plane of symmetry is as follows:

$$\left\langle \left(E_i \vee E_n\right) \wedge \left(E_i \vee E_n \vee H_i \vee H_m\right) \wedge \ldots \wedge \left(E_i \vee E_n\right) \right\rangle \Rightarrow D, \quad (3)$$

where E is an element of the part; *i, m, n*, ... are numerical identification indices of the elements E which take values from an infinity of natural numbers; H is the angular measure of mutual spatial orientation of the elements (this measure is not introduced in the formula provided that the planar elements are arranged in one plane while their axes of symmetry coincide and the elements of the bodies of rotation are in the common axis of symmetry); D is the part.

The structural formula of a part depends on the variant of its composition. All parts are conditionally divided into two sets:

– M_1: flat parts; the parts having the shape of bodies of rotation; the parts of angle and bracket types with axes of the primary elements lying in the same plane or in parallel planes;

– M_2: three-dimensional box-shaped parts and the like; parts of the bracket type in which axes of symmetry of the primary elements lie in intersecting planes.

It is better to arrange the formulas of parts from the M_1 set in a line that emphasizes, in some cases, an analogy with a longitudinal axis or a plane of symmetry of such parts. Formulas of parts of the M_2 set will usually be branched.

The foregoing implicates rules of designing unified parts describing structure of the parts. Here are some of them.

Unification of parts should be based on primary elements and their unified parts. The number of their types should be reduced to one element in each group. The parts should only be composed of primary elements and other unified parts that are regulated by special Lists. It is advisable that the Lists contain tables and drawings of the elements; designation of elements; type of material and coating for various make types; tolerances for linear and angular dimensions; surface roughness, etc. [24]. It is convenient to compile Lists of part elements separately for each type of parts or one-piece assemblies subject to unification (housings, covers, shafts, clamps, angles, etc.), that is for each group of parts. Unified parts may be as follows:

– for angles: diameters of holes, dimensions of grooves, radii of roundings for bending operations;

– for bases of various appliances: wall and bottom thickness, rib thickness, transition radius, holes including threaded holes, step width for detachable parts.

Each part should be assembled from a minimum number of items taken from the List of this group. It is also advisable to unify the Lists.

The elements that broaden fields of their use must be included in the unified parts. For example, grooves should be provided instead of holes in angles to allow their screw connection.

The design and the process of manufacturing unified parts must be capable of obtaining new parts on an accepted unified basis. New parts composed from unified elements should be created only if they are unavailable in the existing bases of unified parts or the parts introduced in production. Quantity of original parts made without the use of basics of group unification should be reduced to the «empty set».

An algorithm of making parts from unified elements can include the following steps:

– define functions that the part should perform;

– search in a corresponding List for the elements that implement functions assigned to the part;

– assemble parts from selected elements;

– check the parts for compliance with the assigned functions.

In the event of discrepancy, the cycle should be repeated with taking into account the new search for unified elements. After solving the basic problems of manufacturing the part and ensuring its functional properties, it is necessary to make drawings or draw the part formula with subsequent preparation of specification. The new unified parts should be entered to the appropriate List. The List should be common for the unified elements and the parts of which it is composed since each part may subsequently be regarded as an element.

Let us construct a formula of a part of the M set (Fig. 2) composed of a primary element (Fig. 1).

In accordance with the standard entries in the design documents, formula of the bracket B is written in the specification as follows:

Bracket. $E+90°+E+E+90°+E$. Designation of the list.

where E is the primary element; 90° is the angle between the elements after bending operation.

In this case, the '+' sign means an additive composition of elements. A reference to a concrete List is required to fully characterize a part since it lists materials, coatings and other requirements depending on the make. It is advisable to present Lists as separate documents. Formula of the part should be written in the Name line and the part designation in the Designation line of the specification. The angle (bracket) B (Fig. 2) is a simple part of the M_2 set. The formula describing the structure will be branched:

Angle. $E+90°+\underset{\underset{E}{+}}{\overset{\overset{E}{+}}{E}}$. The list designation.

It should be noted that the formulas obtained can to some extent serve as an indicator, a criterion of optimality of the part structure. The part can be optimized by means of conversion and simplification of the formula.

Introduction of the presented method should be preceded by preparatory work, mainly preparation of primary Lists and, of course, simplified Lists to begin with. The method makes it possible to change the approach to designing parts, increase productivity of the designer's work, facilitate the important process of improving and expanding the ability to use CAD. When considering unification as a subsystem of standardization, it is logical to introduce its results having a common importance into standards.

5. 3. Formulation of the theorem of design unification

Taking into account the axioms, the presented results of unification and the theorem of existence of designs, new theorems of unification concerning the design structure can be formulated.

In particular, attention is drawn by the fact that introduction of symmetry, e. g. into an element or a part contributes to unification of their structure. Transition to symmetrical elements and parts , extension of symmetry (the symmetry relative to the plane, line or point, i. e. central symmetry) naturally fits into the process of unification. Besides, capability to symmetry is a necessary part of the theory of groups which underlies unification of designs. The above examples are sufficient to inductively prove the theorem of significance or the role of symmetry in unification of the design structure. The task of proving all theorems of the product design is not posed. Besides, inductive proof is probabilistic in nature and no boundary has been found yet where one can assuredly consider that empirical has turned into theoretical in the process of inductive proof.

Therefore, the theorem is presented to some extent hypothetically: capability of primary elements and parts to be symmetrical including what concerns the theory of groups is a necessary condition for their unification by structure.

Since the presence of symmetry is a prerequisite, cases of partial changes in parts should not be considered as an exception to symmetry, for example, introduction of a technological opening, another form of opening for a seal cup, technological boss, etc.

Let us consider the issue of arbitrary transformation of parts (assemblies). It is advisable to divide all parts (assemblies) into two subsets of the total set of parts (assemblies). In the first of these, freedom of transformation of the design structure is limited by strict boundaries of requirements and calculations. These include, for example, designs whose shape must correspond to certain aerodynamic parameters (surface shape of a plane or other flying vehicle fuselage, wing, etc.) or calculations, such as strength when weight exceeds specified weight. The first set also includes designs that have limits concerning ergonomics and safety. The second set contains the rest of designs in which it is possible to perform incomplete but much larger number of structure transformations.

Let us formulate the following theorem for designs of the second set taking into account previous results: a machine or an appliance can be designed in such a way (including recomposition of parts (assemblies)) that any part can converge to one type of the primary element with unified changing sections.

Inductive proof of the theorem is not given there because it is cumbersome. It is more correct to consider this statement as a hypothesis.

Formal notation of the second theorem is as follows:

$$\forall k\left(\begin{pmatrix}\left(k_i,\ldots,k_m,k_n,\ldots\right)\subset M_2\wedge\left(D_i\in k_i\right)\wedge\\ \wedge\left(D_i=\left(E_i\wedge\ldots\wedge E_n\right)\right)\end{pmatrix}\Rightarrow\\ \Rightarrow\left(\left(E_j\wedge\ldots\wedge E_j\right)=D_j\Leftrightarrow D_i\right)\right), \quad (4)$$

where K is general designation of the machine or appliance design; k_i, k_m, k_n are any concrete (i, m, n) designs of machines or appliances; M_2 is the second subset of assemblies and parts included in the total set of assemblies and parts. For assemblies and parts belonging to the M_2, it should be called a set, so that is why both terms were used above; D_i is any part belonging to a corresponding assembly; E_i, E_n, E_j are any concrete primary elements; ⇔ is the symbol indicating equality by definition. It means that any particular D_i part consisting of several types of primary elements is converted to a D_j part consisting of a primary element of one type and both parts are interchangeable by their certain functional parameters.

In the second theorem, the primary elements are considered as ones having sections that allow change of one dimension (for example, section A, Fig. 1).

These theorems can be considered as rules for designing unified parts. These rules should have the status of recommended methods during the period of introduction in manufacture and production tests. After their assimilation in the design and production practice, it is advisable to decide on their mandatory use as a relevant normative document.

5. 4. Studying features of integrated unification of groups of parts and the equipment for their manufacture

Upon considering the issue of group unification of parts (assemblies), it is advisable to study ways of integrated group unification of parts and the equipment for their manufacture.

Comprehensive study of unification of parts and process equipment is consistent with the system approach and gives grounds to expect useful practical results.

In order to achieve an economic effect, technological processes, equipment and tools must have a possibility of group processing and rapid readjustment for elementwise manufacture of unified parts. However, a series of questions arise:

– which, from theoretical and practical point of view, formalized congruencies between parts and equipment should be ensured for optimality of decisions?

– whether it is possible to make any number of parts, for example, parts with small dimensions in one variant or with equipment of one design that will fully satisfy mathematical requirements concerning the group?

Let us consider the method of group processing [22] and, in particular, a composite part by turning. Composite part is the starting theoretical and applied concept in development of group technological processes. In lathe machining, due to rotation of the workpiece and wide range of choice of tool trajectory, a considerable range of parts can be processed. As for a composite part, it is possible to obtain its components through usual adjustment of lathes which results in an improved economic effect.

This point is interesting from the standpoint of mathematical theory of groups since composite parts are rightly considered a group. Depending on complexity, the parts forming the composite part are its subgroups or elements. In this case, we should mention the concept of isomorphism of groups. Two groups are called isomorphic if there are a mutual one-to-one mapping of one group to the other and a stored operation. Isomorphism expresses abstract identity of structures of two groups. When considering a composite part, it is necessary to note that there is an isomorphic correspondence between its shape and elements of the lathing process (workpiece rotation, working movements of cutters). The presented example demonstrates optimal conditions for group unification due to the process specificity and the composite part that is a body of rotation. Presence of isomorphism between the design and technological issues can be an indicator of maximum level of integrated unification of a group of parts and equipment for their production. This conclusion applies to the section generated by isomorphic mapping.

Compared to turning, achievement of isomorphism between a group of unified parts of other types, for example, parts punched from a sheet metal, cast, pressed, forged and one concrete piece of equipment for their manufacture is a difficult task.

In particular, let us consider the process of manufacturing a group of cold-punched parts in a reconfigurable die of sequential action. Strictly speaking, isomorphism between parts and tools is missing for all or most of the group parts. This is because the die elements corresponding to various parts and their elements are dispersed in the die, therefore, in a mathematical sense, operation is not preserved. Thus, if any part is an additive composition, then its image in the tool will not be the additive composition in this case. Absence of isomorphism caused by distribution of operations in the die is involuntarily associated with the worse conditions for obtaining precise parts. Unfortunately, such conditions take place. Theoretically, by analogy with turning, isomorphism in this case can be achieved if a group of element-wise unified parts is formed by their imposition. It is imperative to ensure a concordant arrangement of primary elements. Manufacture should be carried out in a special reconfigurable die of combined action. The number of parts in these groups is insignificant for the current level of die development. There are ways to improvement:

– design and manufacture interchangeable unified elements for subgroups of parts and then apply group processing according to [22] using these elements;

– use composite dies of combined and sequential action;

– in the future, it is possible to use dies that are quickly readjusted due to, e. g. magnetic change of shape of the working elements.

When casting into molds, mapping of the workpiece into corresponding mold planes will be isomorphic. In this case, the possibility of making a group of elementwise unified parts in a single mold is connected with forced and inevitable limitation of their nomenclature. This problem can be solved applying the following measures:

– discreetly change the part dimension in the direction of movement of the movable section of the mold and relate this adjustment to extension of the part nomenclature;

– combine necessary machining with elimination of the elements that are unnecessary for the given part;

– make mold inserts for subgroups of parts using all interchangeable elements by analogy with the group method [22].

Of course, this approach complicates designing groups of unified parts. In addition, there are specific complexities of the foundry technology: shrinkage, probability of crack formation, warpage after machining, etc. However, it is possible to eliminate these difficulties.

It should be noted that refinement concerning obtaining of a workpiece or a part as a result of the operation is only possible in cases where it is of fundamental importance in theoretical sense.

Integrated unification of groups of elementwise unified parts and equipment for their manufacture consists in the following: unification is technologically implemented by a method in which the whole group is completely (or to a large extent) made in equipment of one specific execution which results in reduction of the equipment nomenclature.

When considering unification of technological parts of equipment, unification of its structural parts (blocks, packs, etc.) which is presented in [22] was not taken into account.

Without claiming existence of a regularity consisting in a necessity of isomorphism to achieve integrated group elementwise unification of parts and equipment, we shall restrict ourselves to formula construction. The following formula is syntactically identical. However, assertion of its universal significance is possible only after the proof of semantic truthfulness. Inductive proof is not given there because of its cumbersome nature.

$$\left(\left(f : G^W \to T^{equip}\right) \wedge I^U\right) \Rightarrow (I^U = \max), \qquad (5)$$

where f is the symbol of mapping, it is isomorphic in this case; G^W is the group of unified parts (workpieces); T^{equip} is the concrete unit of technological equipment intended for G^W manufacture; I^U is an indicator of unification.

When considering this issue, two technological trends emerging in present-day technology and can be distinguished by the place of their unification should be pointed out. According to the first trend, there is no need to force unification since shapes of the parts can vary as a result of applying a technology that has effectively replicated human features acceptable for production. It can be robots and devices with technical vision, artificial intelligence, etc. Shape of the parts can vary with application of a technology that uses significant improvements in equipment including improvements in various physical principles. A great deal of variety can be achieved in shaping parts by combining discrete and continuous shaping in a multivariant way.

According to the second trend, unification should be given a paramount importance. It is likely that structure of the part (assembly) designs by the type of the mathematical theory of groups will be predominant using an appropriate technology that uses rotor lines, machining centers and other types of technological equipment. Due to its peculiarities, this equipment is more suitable for discreetness of part changes without exclusion of monotonic, continual changes, accordingly.

The mentioned trends are interpenetrating and complement each other in the common technological process. Their isomorphism study is sufficiently informative and can be used to determine device identity, group equivalence and other features with an aim of unification of model construction and the like.

6. Discussion of results obtained in studying features of integrated unification of groups of parts and equipment for their manufacture

As a result of the study, a formal description of primary element (1) as the basis of a part or a group of parts and a mathematical expression (2) reflecting group properties of designs were presented. For an sample, a structural design of the primary element (Fig. 1) and examples of composing parts from primary elements (Fig. 2) were given. It was established that the primary element (if all its dimensions are fixed) is the basic element of a group of parts, and this group is cyclic, periodic and commutative in the case of a centrally symmetric primary element. This approach has opened up the possibility of using the theory of groups to formalize generation of unified parts and design them using the CAD system.

A formula of unified parts (3) was obtained which can serve as a criterion of optimality of the part structure. According to this formula, an algorithm of obtaining a new part from unified elements was constructed and rules for designing unified parts were written (p. 5. 2). This method changes approach to designing parts and provides high manufacturability of designs.

Theorems of design unification (p. 5. 3) were derived. They can be considered as rules for designing unified parts. The high general importance of unification theorems for the design of process appliances and machines allows us to consider them as laws. According to provisions of the unification theorems, a formalized entry (5) for integrated unification of groups of elementwise unified parts and equipment for their manufacture (p. 5. 3) was presented.

Use of unified parts and assemblies will have a positive impact on time of product development not only within one model line but also with other related and nonrelated trends and destinations. Not only unification of technological elements but also unification of the design properties of these elements is the base of unification. Thus, the use of these elements accelerates design and reduces production of unique design objects to a mere assembly of commercially available elements. As a result, a significant reduction in the cost of such products is obtained which improves competitiveness of the final product. The use of unified assemblies (unification of a higher level) will result in designing products on a modular basis. Such an approach is one of the major current trends in designing products for engineering and the industry in general.

One of limitations of widespread use of the integrated group unification of designs can be caused by constant evolution of technological processes including shaping operations. With the rapid development of 3D printing parts of various materials, the unification theory will require development and adaptation to the latest technologies. The process of designing with an extended application of the integrated unification will require a more thorough theoretical training of specialists, increasing the level of operational knowledge, studying the interface and capabilities of software. In addition, use of a wide range of unified parts is only advisable within the products of one manufacturer since most manufacturers earn up to 60 % of their profits in production of spare parts for their own products. Thus, manufacturers are interested in certain design features of their products which will make it possible to «tie» their consumers. The widespread free dissemination of bases of 3D models of elements, parts and modules is an important and effective way for designers to promote their products in engineering markets.

Application of the unification formulas (1) to (5) will have the best effect on designing when using modern CAD systems and manufacturing products of similar purpose, for example, conveyors, roller tables, etc. With significant differences between the input design parameters, such designs can be easily structured and broken down into single, easy to unify elements. This makes the process of designing and production similar to the design process with the Lego Designer. Potential profitability from implementation of integrated unification will be determined by the time spent on designing. An object of medium complexity which has a prototype is designed within 1–2 months. In contrast, design of an object which has no unified elements takes 6 months or more. Application of the integrated group unification can several times cut time of designing structures which on average will reduce cost of design work by 30 % or more.

Further studies in this area should be focused on the process of automating creation of a unified set of elements based on an innovative conceptual design.

7. Conclusions

1. A concept of primary element was considered and the method of its construction and mathematical description were presented for the group unification of designs of parts (assemblies) of machines and appliances. It was proved that application to the set of parts (assemblies) and their elements the law of composition, introduction of operations similar to algebraic ones, use of the concept of isomorphism, etc. allows one to formalize and put in order the unification process and increase its efficiency. It was also proved that the primary element (provided that its dimensions are fixed) is a generating element of the group of parts and the group itself is cyclic, periodic and (with a centrally symmetric primary element) commutative. The obtained results open up possibilities for using apparatus of the theory of groups to formalize creation of unified parts and charge their design to a CAD system.

2. Using the theory of groups, formulas of unified parts were derived. These formulas can, to some extent, serve as an indicator or a criterion of optimizing the part structure: the part can be optimized by transformation and simplification of the formulas. However, effective implementation of these formulas is preceded by a considerable amount of preparatory work, mainly compilation of initial simplified Lists of properties of parts and unified parts. The method makes it possible to change the approach to designing parts (assemblies), accelerate the design process with the help of a CAD system, improve work productivity.

3. Theorems of unification of part (assembly) design structure were formulated taking into account axioms, results of unification and the theorem of existence of designs concerning the design structure proper. The mentioned theorems can be considered as rules for designing unified parts. The high general significance of the above theorems of unification of design of process appliances and machines makes it possible to consider them as laws. Moreover, the derived formula of the theorem of design unification and its formalized description are valid for any set of designs. Following the period of development and testing in production, these rules should obtain status of recommended methods. After their assimilation in the design and technological practice, it is advisable to resolve the issue of their mandatory application with an issue of an appropriate regulatory document.

4. Peculiarities of integrated unification of groups of parts and the equipment for their manufacture were studied. Formula of integrated group unification of parts and equipment was presented and its semantic truthfulness was proved. The examples show that for many types of parts and assembly units, formulas can be compiled instead of making drawings which, in particular, can serve as a prerequisite for creation of customizable process equipment. Compilation of formulas instead of working drawings is an effective means of simplifying designs and introduction of unification elements. Presence of design isomorphism in terms of any technological operation (process) is an indicator of prospective manufacturability of parts and assemblies.

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