LIMIT BEHAVIOR OF EXPECTED ESSCHER TRANSFORM

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Esscher transform of random variable X with parameter $\alpha \in \mathbb{R}$ is a random variable defined in the following way

$$ET[X;\alpha] := \frac{Xe^{\alpha X}}{\mathsf{E}[e^{\alpha X}]}.$$

Esscher transform was introduced by Swedish mathematician Fredrik Esscher in the paper [2]. Results of the just mentioned paper were later extended in the paper [3].

Expected Esscher transform of a random variable X with parameter $\alpha \in \mathbb{R}$ is defined as expectation of corresponding Esscher transform, namely

$$EET[X;\alpha] := \mathsf{E}[ET[X;\alpha]] = \frac{\mathsf{E}[Xe^{\alpha X}]}{\mathsf{E}[e^{\alpha X}]}.$$

Expected Esscher transform with parameter $\alpha \geq 0$ is often used as a method of pricing of insurance contracts (in this case variable X indicates size of insurance compensation related to some insurance pact) and is called in actuarial literature Esscher premium. Application of Esscher transform to pricing of insurance contracts probably was initiated by Swiss mathematician Hans Bühlmann in the paper [1].

Review of known results associated with Esscher transform can be found, for example, in the work [4].

We will also need the following two definitions:

essential supremum of random variable X with distribution function $F(x)$

$$\operatorname{ess\,sup}[X] := \sup\{\delta\colon F(\delta) < 1\};$$

essential infimum of random variable X with distribution function $F(x)$

$$\operatorname{ess\,inf}[X] := \inf\{\delta\colon F(\delta) > 0\}.$$

Limit behavior of expected Esscher transform is following.

Theorem 1. *For any random variable X holds limit relation*

$$\lim_{\alpha\to+\infty} EET[X;\alpha] = \operatorname{ess\,sup}[X].$$

Theorem 2. *For any random variable X holds limit relation*

$$\lim_{\alpha\to-\infty} EET[X;\alpha] = \operatorname{ess\,inf}[X].$$

References

[1] H. Bühlmann, *An economic premium principle*, ASTIN Bull. **11** (1980), no. 1, 52–60.

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[3] F. Esscher, *On approximate computations of distribution functions when the corresponding characteristic functions are known*, Skand. Aktuarietidskr. **46** (1963), 78–86.

[4] H. Yang, *Esscher transform*, Encyclopedia of Actuarial Science, John Wiley & Sons, 2004, 617–621.

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